TONBRIDGE SCHOOL

Scholarship Examination 2017

MATHEMATICS II

Wednesday 26th April 2017
2.00 pm

Time allowed: 1 hour 30 minutes

There are seven questions in this paper
Answer as many questions as you can.
All the questions carry equal marks.
You may attempt the questions in any order.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

1. Excited hydrogen atoms emit radiation with particular wavelengths, L nanometres, given by the formula $\frac{91}{L} = \frac{1}{4} - \frac{1}{n^2}$ where n is a whole number greater than 2.

(a) If $n = 7$, show that $L = 396$ (correct to 3 significant figures).

(b) If $L = 433$, what is the value of the whole number n?

(c) Find the values of n corresponding to visible light for which L is between 380 and 780.

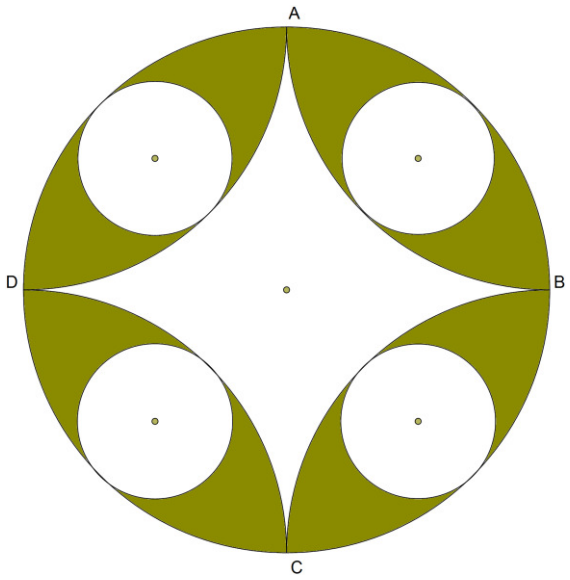
(d) If n is very large, what is the value of L?

2. Solve the following set of simultaneous equations to find the values of x, y and z.

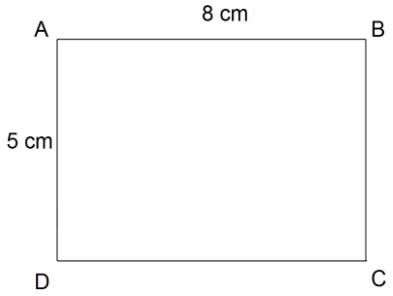
$$5x + 3y + 3z = 10$$
$$3x + 5y + 3z = 4$$
$$3x + 3y + 5z = -14$$

3. In the symmetrical figure below, the big outer circle has radius 10 cm. The shaded regions are all identical and symmetrical about the lines AB, BC, CD and DA. The small circles are in the middle of these regions and touch the sides.

Making your method clear, find the total shaded area in the figure.



4. A solid pyramid has a 5 cm by 8 cm rectangular base $ABCD$ with the dimensions shown in the diagram. The vertex (top point) of the pyramid, V, is directly above the centre of the base with $VA = VB = VC = VD = 7$ cm.

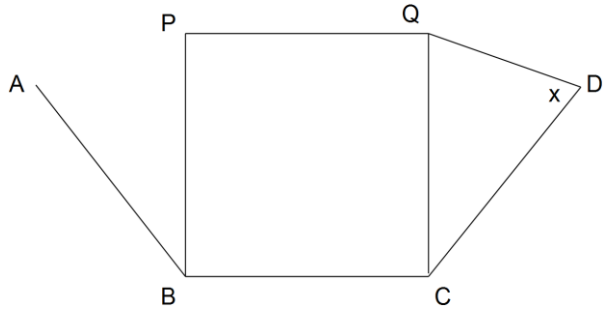


(a) Draw a sketch of the pyramid.

(b) Find the total surface area of the four triangular faces and the base.

(c) Find the area of triangle AVC, which is a slice through the pyramid.

5. In the diagram below, $ABCD$ are four successive vertices of a regular polygon. $BPQC$ is a square and angle $C\hat{D}Q$ is x degrees.



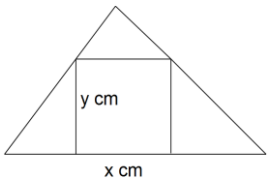
(a) Find x if $ABCD$ are vertices of an 8-sided polygon.

(b) How many sides does the polygon have if $x = 57°$?

(c) What is the smallest possible value of x?
(Explain your answer carefully.)

TURN OVER

6. This question is about triangles with a fixed area of 6 cm^2. If the base of the triangle has length x cm, the height y cm of the largest square that fits inside the triangle as shown in the diagram is given by the formula $y = \dfrac{12x}{x^2 + 12}$.



(a) Make a table of y values (correct to 2 decimal places) for x values starting at $x = 0$ and going to $x = 5$ in steps of 0.5.

(b) Using your values in (a) and choosing sensible scales, plot a graph of y against x.

(c) (i) What is the greatest possible area of the square?
(ii) What is the height of the triangle when this occurs?

7. Study the pattern of numbers in the table below carefully. Column B gives the sum of the numbers in Column A.

	A	**B**
Row 1	1	1
Row 2	3 + 5	8
Row 3	7 + 9 + 11	27
Row 4		
Row 5		
Row *n*		

(a) Write down the entries in Columns A and B for Row 4 and Row 5.

(b) For Row n, find formulae in terms of n for :
(i) The entry in Column B;
(ii) The number of terms in Column A;
(iii) The mean (average) of the terms in Column A.

(c) Find the value n if the sum in Column A of Row n starts with 4423 and ends with 4555.

(d) If the entry in Column B is 185193, what is the smallest number in the sum in Column A?

(e) If the entry in Column B is 314432, what is the largest number in the sum in Column A?

END OF PAPER